On a Competitive Secretary Problem

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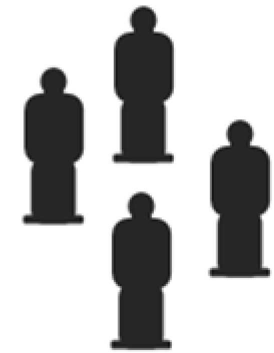
Motivating problem

- Suppose multiple employers are competing to hire the best possible employee:
  - Each university wants to hire the best candidate(s) they can
  - Given a choice, candidates are likely to choose higher-ranked universities over lower-ranked ones

Motivating problem

Multiple employers are competing to hire the best possible employee

Universities



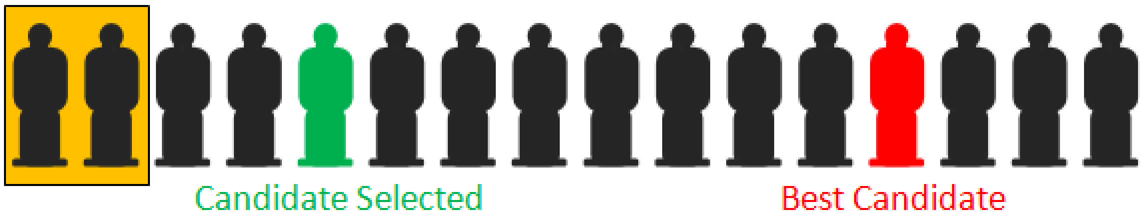
Faculty Applicants

How does the competition between the employers affect their hiring strategies and their ability to hire the best possible candidates?

This paper

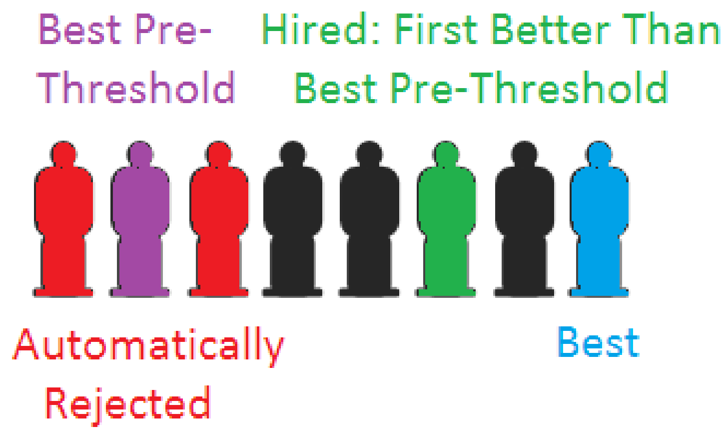
- A stylized model for competitive hiring
  - Optimal strategies of the employers in equilibrium
  - Use these to understand how well the employers and applicants do in this competitive setting.
- Our model: a generalization of the classic secretary problem.

Secretary Problem



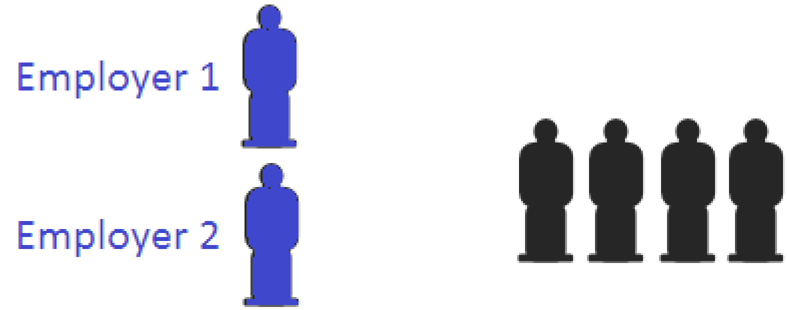
- A single employer wishes to hire the best candidate out of n totally ordered candidates.
  - Candidates arrive in random order
  - Information only on seen candidates
  - Immediately after arrival, employer makes irrevocable hiring decision
- Goal: maximize probability of hiring best

Secretary Problem



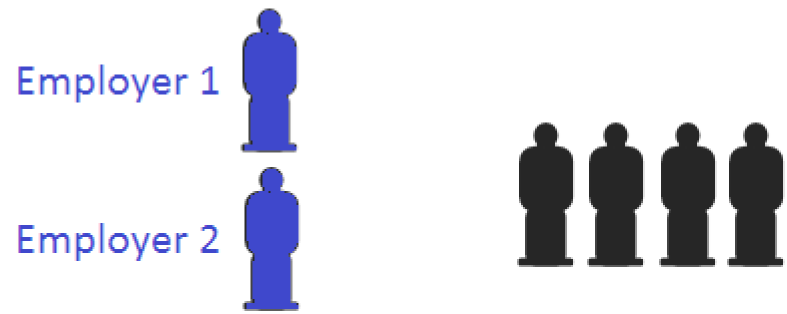
- Threshold strategy :
  - Reject T candidates; remember the best of them, say B
  - After threshold, hire first candidate better than B.
- The optimal strategy is threshold n/e in the limit, with probability of success $1/e$.

Our model: competitive secretary



- k ranked employers
- n ranked applicants arriving in random order.
- When applicant arrives, each employer chooses whether or not to make offer.
  - Applicant accepts the offer of the highest ranked employer she receives an offer from.
  - Once an employer has hired, no more offers.

Model: competitive secretary



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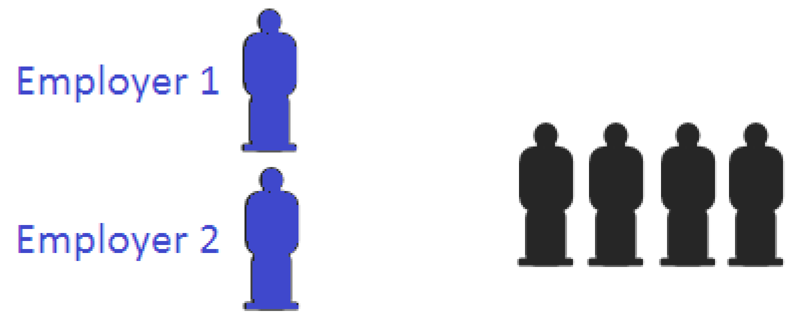
- Goal of each employer: maximize the probability of hiring the very best applicant

Example: 2 employers



- Goal of each employer:

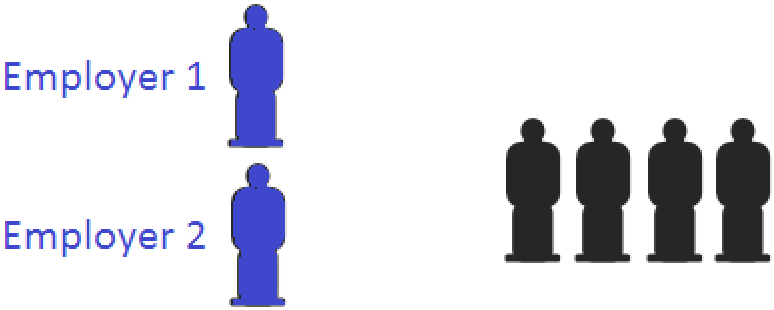
  maximize the probability of hiring the very best applicant

- Strategy for 1:
  - regular optimal secretary!
- Strategy for 2:
  - must take into account fact that after n/e, will lose out on first simultaneous offer.

Optimal strategy:

Threshold strategy with $T = e^{-\frac{3}{2}}n$

Example: 2 employers



- Optimal strategy for 1:
  - Threshold strategy with $T = e^{-1}n$
- Optimal strategy for 2:
  - Threshold strategy with $T = e^{-\frac{3}{2}}n$

- This pair of strategies is **a subgame-perfect Nash equilibrium** in this game.
  - Each employer's strategy is a best response to other's at all times during the game.

Optimal Strategies can be computed by dynamic programming

$$R_k(i-1) = \frac{1}{i}\min\{R_k(i), 1-\frac{i}{n}\} + (1-\frac{1}{i})R_k(i)$$
$$R_k(i-1) = \frac{1}{i}R_{k-1}(i) + (1-\frac{1}{i})R_k(i)$$

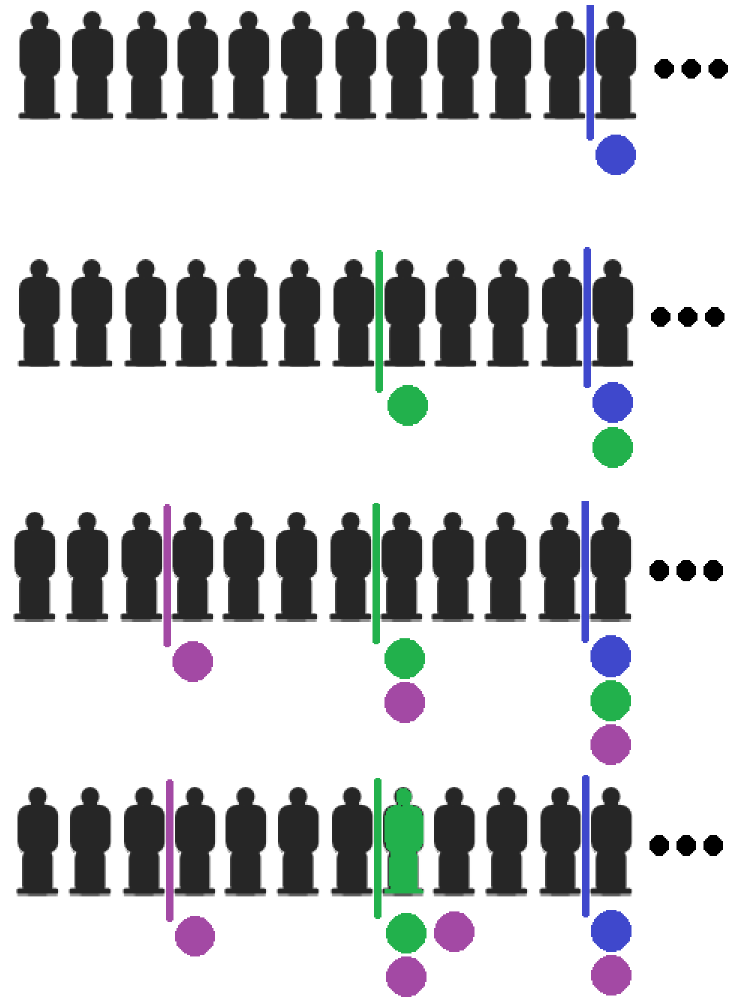
- Theorem: For Employer j the optimal and subgame perfect strategy is threshold strategy T_j, where

$$T_j = \min\{i : R_j(i) \geq 1 - \frac{i}{n}\} - 1.$$

  - Also, $T_j \geq T_{j+1}$

Illustration of Optimal Strategies

- Employer 1's view:
- Employer 2's view:
- Employer 3's view:
- Employer 3's view if Employer 2 hires



Computing optimal thresholds

- $O(kn)$ algorithm for numerical computation
- Closed form for k small.

Theorem:

Probability of success = Threshold

$$\lim_{n\to\infty} Pr\,(j \text{ hires best}) = t_j := \lim_{n\to\infty} \frac{T_j}{n}.$$

(not intuitive)

Employer Rank	Threshold ($n \to \infty$)
1	.368
2	.223
3	.141
4	.091
5	.059
6	.039
7	.026
8	.017
9	.012
10	.008

Conclusion

- Problem of multiple ranked employers trying to hire the best applicant (or one of top k)
- Best strategies are threshold strategies
  - The lower the rank of the employer, the earlier they will start making offers
- Many interesting open questions:
  - What if employers can hire a past candidate and applicants are strategic, hold out for better offers?
  - How do equilibria change when salaries are introduced?

Thank you!

Q & A

Partial References

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Solution via Dynamic Programming

- If $i \le T_{k-1}$,

$$R_k(i-1) = \frac{1}{i}\min\{R_k(i), 1 - \frac{i}{n}\} + (1 - \frac{1}{i})R_k(i)$$

- Otherwise,

$$R_k(i-1) = \frac{1}{i}R_{k-1}(i) + (1 - \frac{1}{i})R_k(i)$$

- $R_k(i)$ is the risk of rejecting candidate i
- $1 - i/n$ is the risk of accepting candidate i

Intuition of Optimal Strategy

- Consider i–th candidate if they are best so far
  - $R_j(i)$ if reject; $1 - i/n$ if accept